Math4230 Tutorial 1

1. True or False (Please give a proof or explanation if it is true and give a counterexample if it is false)
   (a) Any affine set M in $\mathbb{R}^n$ contains the zero vector;
   (b) Any affine set M in $\mathbb{R}^n$ contains a nonzero vector;
   (c) The union $L \cup M$ of two given affine sets in $\mathbb{R}^n$ is an affine set;
   (d) Hyperplane ($\{x : a^T x = b, a, x, b \in \mathbb{R}^n\}$) is affine set.

2. Is the set $\{a \in \mathbb{R}^k \mid p(0) = 1, |p(t)| \leq 1 \text{ for } \alpha \leq t \leq \beta\}$ convex? where
$$p(t) = a_1 + a_2 t + \cdots + a_k t^{k-1}, \quad a = (a_1, a_2 \cdots, a_k)^T$$

3. Show that the following sets are convex respectively,
   (a) $A = \{(x, t) \in \mathbb{R}^{n+1} \mid \|x\| \leq t, x \in \mathbb{R}^n, t \in \mathbb{R}\}$;
   (b) $B = \left\{x \in \mathbb{R}^n | \ (x - x_c)^T P^{-1}(x - x_c) \leq 1\right\}$, where P is symmetric positive definite and $x_c \in \mathbb{R}^n$;
   (c) $C = \{x \in \mathbb{R}^n | \ \|x - x_c\|_2 \leq r, x^T x_b > 0\}$, where $r > 0$ and $x_c, x_b \in \mathbb{R}^n$. Assume that C is not a null set.